

# Four-Color Matrix Method for Correction of Tristimulus Colorimeters

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## Abstract

A new calibration method has been developed to improve the accuracy of chromaticity coordinates obtained from a tristimulus colorimeter for color displays. Matrix methods such as the one recommended by ASTM are well known for this purpose, but they may fail to work as expected due to experimental noise and errors. As these matrix methods are based on tristimulus values, the accuracy of the luminance measurement affects the accuracy of the corrected chromaticity. This new method utilizes  $x$ ,  $y$  values only, and is independent of  $Y$  values. Thus, in principle, it eliminates errors due to luminance measurement variations. A correction matrix is obtained from the  $x$ ,  $y$  values of three primary colors and a white color of a display, measured by the target instrument and a reference instrument. A computer simulation was conducted to evaluate the effect of random noise in  $Y$ . Experiments were conducted using a commercial tristimulus colorimeter and a spectroradiometer, measuring 14 colors of a CRT display. The results show noticeable improvement in chromaticity accuracy over the current practice.

## Introduction

Accurate chromaticity measurements of color displays such as cathode ray tubes (CRTs) and flat panel displays are increasingly important as their qualities improve and customers demand accurate color reproduction. Tristimulus colorimeters are commonly used to measure chromaticity of such displays. However, due to imperfect matching of the spectral responsivities of tristimulus colorimeters to the color matching functions, measurement errors are inevitable when the spectral power distribution of a display being tested is dissimilar to that of the calibration source. Tristimulus colorimeters and luminance meters are normally calibrated with CIE Illuminant A.

Matrix techniques are known to improve the accuracy of tristimulus colorimeters for color display measurements, utilizing the fact that the colored light produced by most displays is a linear superposition of the spectral power distributions of three primaries. ASTM (American Society for Testing and Materials) E1455 [1,2] recommends a method to derive a correction matrix ( $R'$  matrix) that transforms measured values into better agreement with the reference values  $X$ ,  $Y$ ,  $Z$ . The matrix is made such that the root-mean-square difference between transformed and  $X$ ,  $Y$ ,  $Z$  for several different colors of a display is minimized.

These matrix methods, however, may not work as expected due to experimental noise and errors. Because these conventional methods are based on tristimulus values, the errors in luminance affect the accuracy of the corrected results of chromaticity as well as luminance. The variation of luminance measurements can occur due to instability of the display, flicker effect on the detectors, interreflections between the display surface and the instrument, etc., while the measurement of  $Y$  is normally more stable and reproducible since it is a relative measurement, and the error factors mentioned above tend to be canceled out if the three channels are sampled at the same time. This problem can be solved by our recent work [3], in which the correction matrix is determined by minimization only for differences. This method, however, requires a numerical iterative solution, and it is difficult to apply for a portable instrument.

A new technique for the matrix method (named Four-Color Method) has been developed, which is based on the  $(x, y)$  values only, and is independent of  $Y$  value. Thus, in principle, it eliminates errors arising due to luminance measurement variations. The correction matrix is obtained from the  $(x, y)$  values of the three primary colors plus white from the display measured by a target instrument and a reference instrument. A computer simulation was conducted to evaluate the

effect of random errors in  $Y$ . An experiment was conducted using a commercial tristimulus colorimeter and a spectroradiometer, measuring 14 colors of a CRT display. The results are analyzed using the Four-Color Method as well as other conventional methods.

## 2. THEORY

The primary colors (red, green, and blue) and a white color of a display are measured by a target instrument (a colorimeter being optimized) and a reference instrument (a reference tristimulus colorimeter or spectroradiometer). From the chromaticity coordinates  $(x_{m,R}, y_{m,R})$ ,  $(x_{m,G}, y_{m,G})$ , and  $(x_{m,B}, y_{m,B})$  of red, green, and blue measured by the target instrument, the relative tristimulus values of the primary colors from the target instrument are defined by

$$\begin{aligned} \mathbf{M}_{\text{RGB}} &= \begin{bmatrix} X_{m,R} & X_{m,G} & X_{m,B} \\ Y_{m,R} & Y_{m,G} & Y_{m,B} \\ Z_{m,R} & Z_{m,G} & Z_{m,B} \end{bmatrix} \\ &= \begin{bmatrix} x_{m,R} & x_{m,G} & x_{m,B} \\ y_{m,R} & y_{m,G} & y_{m,B} \\ z_{m,R} & z_{m,G} & z_{m,B} \end{bmatrix} \begin{bmatrix} k_{m,R} & 0 & 0 \\ 0 & k_{m,G} & 0 \\ 0 & 0 & k_{m,B} \end{bmatrix} \quad (1) \end{aligned}$$

$$\text{where } k_{m,R} + k_{m,G} + k_{m,B} = 1.$$

$k_{m,R}$ ,  $k_{m,G}$  and  $k_{m,B}$  are the relative factors for measured luminance of each display color, and are now unknown variables.  $z$  with any subscript  $s$  is obtained from  $x_s$  and  $y_s$  by

$$z_s = 1 - x_s - y_s. \quad (2)$$

From the chromaticity coordinates  $(x_{r,R}, y_{r,R})$ ,  $(x_{r,G}, y_{r,G})$ , and  $(x_{r,B}, y_{r,B})$  of red, green, and blue measured by the reference instrument, the relative tristimulus values of the primary colors from the reference instrument are defined by

$$\begin{aligned} \mathbf{N}_{\text{RGB}} &= \begin{bmatrix} X_{r,R} & X_{r,G} & X_{r,B} \\ Y_{r,R} & Y_{r,G} & Y_{r,B} \\ Z_{r,R} & Z_{r,G} & Z_{r,B} \end{bmatrix} \\ &= \begin{bmatrix} x_{r,R} & x_{r,G} & x_{r,B} \\ y_{r,R} & y_{r,G} & y_{r,B} \\ z_{r,R} & z_{r,G} & z_{r,B} \end{bmatrix} \begin{bmatrix} k_{r,R} & 0 & 0 \\ 0 & k_{r,G} & 0 \\ 0 & 0 & k_{r,B} \end{bmatrix} \quad (3) \end{aligned}$$

$$\text{where } k_{r,R} + k_{r,G} + k_{r,B} = 1.$$

$k_{r,R}$ ,  $k_{r,G}$  and  $k_{r,B}$  are the relative factors for luminance of each display color.

Based on the additivity of tristimulus values, and with  $(x_{m,W}, y_{m,W})$  and  $(x_{r,W}, y_{r,W})$  being the chromaticity coordinates of the display for the white color measured by the target instrument and the reference instrument, respectively, the following relationships hold:

$$\begin{bmatrix} x_{m,W} \\ y_{m,W} \\ z_{m,W} \end{bmatrix} = \begin{bmatrix} x_{m,R} & x_{m,G} & x_{m,B} \\ y_{m,R} & y_{m,G} & y_{m,B} \\ z_{m,R} & z_{m,G} & z_{m,B} \end{bmatrix} \begin{bmatrix} k_{m,R} \\ k_{m,G} \\ k_{m,B} \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} x_{r,W} \\ y_{r,W} \\ z_{r,W} \end{bmatrix} = \begin{bmatrix} x_{r,R} & x_{r,G} & x_{r,B} \\ y_{r,R} & y_{r,G} & y_{r,B} \\ z_{r,R} & z_{r,G} & z_{r,B} \end{bmatrix} \begin{bmatrix} k_{r,R} \\ k_{r,G} \\ k_{r,B} \end{bmatrix} \quad (5)$$

The white color of the display can be of any intensity combination of the three primary colors. The values  $(k_{m,R}, k_{m,G}, k_{m,B})$  and  $(k_{r,R}, k_{r,G}, k_{r,B})$  are now obtained by solving Eqs.(4) and (5) as

$$\begin{bmatrix} k_{r,R} \\ k_{r,G} \\ k_{r,B} \end{bmatrix} = \begin{bmatrix} x_{r,R} & x_{r,G} & x_{r,B} \\ y_{r,R} & y_{r,G} & y_{r,B} \\ z_{r,R} & z_{r,G} & z_{r,B} \end{bmatrix}^{-1} \begin{bmatrix} x_{r,W} \\ y_{r,W} \\ z_{r,W} \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} k_{m,R} \\ k_{m,G} \\ k_{m,B} \end{bmatrix} = \begin{bmatrix} x_{m,R} & x_{m,G} & x_{m,B} \\ y_{m,R} & y_{m,G} & y_{m,B} \\ z_{m,R} & z_{m,G} & z_{m,B} \end{bmatrix}^{-1} \begin{bmatrix} x_{m,W} \\ y_{m,W} \\ z_{m,W} \end{bmatrix} \quad (7)$$

The correction matrix  $\mathbf{R}$  is then given by

$$\mathbf{R} = \mathbf{N}_{\text{RGB}} \mathbf{M}_{\text{RGB}}^{-1}. \quad (8)$$

If the relative tristimulus values  $\mathbf{M}$  for any colors measured by the target instrument are given by

$$\mathbf{M} = \begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = k \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} \quad (9)$$

where  $k$  is an arbitrary factor, then the relative tristimulus values  $\mathbf{M}$  from the target instrument are corrected to  $\mathbf{M}$  by using the correction matrix  $\mathbf{R}$  as

$$\mathbf{M} = \mathbf{R} \mathbf{M}. \quad (10)$$

The corrected chromaticity coordinate  $(x, y)$  is computed from  $\mathbf{M}$ .

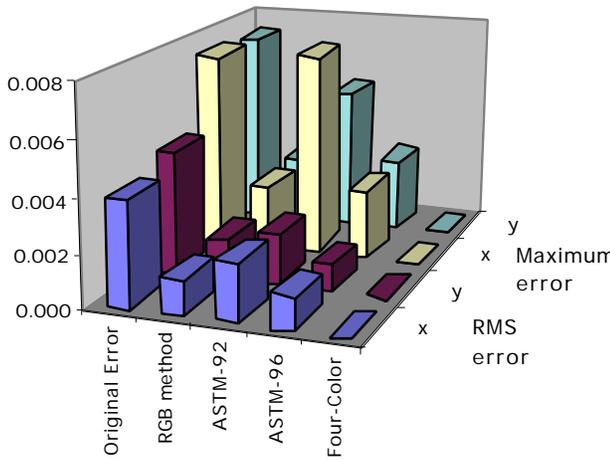


Fig.1. Errors (  $x$ ,  $y$  ) for the 16 CRT colors, with 1.7 % rms random noise in  $Y$ , after correction by each method.

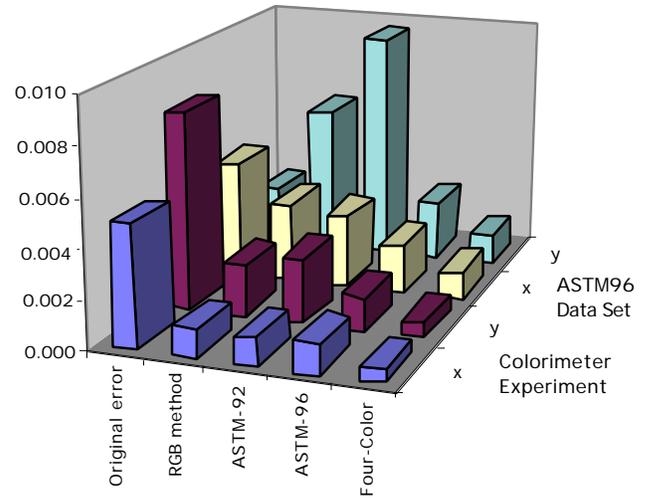


Fig. 2 : RMS differences (  $x$ ,  $y$  ) for all the CRT colors after corrections.

### 3. SIMULATION

In order to evaluate how errors in luminance  $Y$  may affect the errors in chromaticity ( $x$ ,  $y$ ), a computer simulation was carried out using the spectral responsivity data of a real tristimulus colorimeter and the spectral power distributions of the primary colors of a real color CRT display. CIE  $f_1'$  values of the colorimeter with respect to  $\hat{x}(\lambda)$ ,  $\hat{y}(\lambda)$ , and  $\hat{z}(\lambda)$  were 7.6 %, 3.8 %, and 7.7 %, respectively. The model colorimeter was first calibrated against CIE Illuminant A. The intensity scales of red, green, and blue were adjusted so that a mixture of these primary colors, each at 100 % intensity, created a white color of approximately 9000 K. Then, 16 different colors were created by different intensity combinations of the primary colors. The values of  $x$ ,  $y$ , and  $Y$  of each color as measured by the colorimeter, as well as their true values, were calculated. Then, random noise of 1.7 % rms (3 % maximum) were imposed on  $Y$ . This does not affect  $x$ ,  $y$  values. Table 1 shows these calculated chromaticity coordinates and luminance values of the 16 CRT colors. The random noise values in the table are an example because they are randomized everytime the calculation is run.

These  $x$ ,  $y$ , and  $Y$  values (with random noise) were converted into tristimulus values, and the corrected tristimulus values (and the chromaticity coordinates) were computed using (1)  $R$  matrix in ASTM E1455-92

[1] (denoted as RGB method), (2)  $R'$  matrix in ASTM E1455-92 [1] (denoted as ASTM-92), (3)  $R'$  matrix in ASTM E1455-96 [2] (denoted as ASTM-96), and (4) the Four-Color Method. Figure 1 shows the results of the simulation. The bars show the rms and the maximum of the errors ( $x$ ,  $y$ ) for all the 16 colors after correction by each method. These results were obtained as an average of 10 repetitions of the entire computation since there were variations in the results at each randomization of the  $Y$  noise. These results verify that luminance errors significantly affect chromaticity values with the conventional methods (especially ASTM-92). The Four-Color Method is not at all affected by luminance errors.

### 4. EXPERIMENT

To verify the effectiveness of the proposed method, an experiment was conducted using a commercial tristimulus colorimeter as the target instrument and a scanning-type spectroradiometer (employing a double monochromator) as the reference instrument. A video signal generator created 14 different colors on a broadcast-quality color CRT. After changing to a new color, the monitor was allowed to stabilize for  $\sim 2$  min. Both instruments were equipped with lens systems that collected light from the center area of  $\sim 3$  cm diameter on the screen. Measurements took place with each instrument

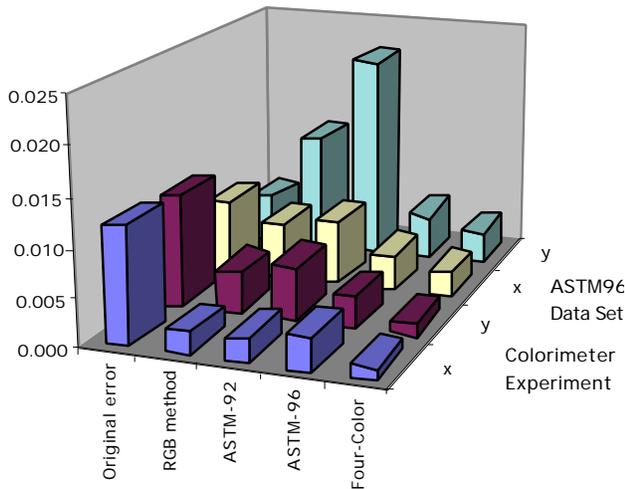


Fig. 3: Maximum differences ( $x$ ,  $y$ ) for all the CRT colors after corrections.

sequentially. All measurements were taken in a darkened environment. Table 2 shows the data of chromaticity coordinates and luminance values measured by each instrument.

The reference spectroradiometer was calibrated against a spectral irradiance standard lamp used with a calibrated polytetrafluoroethylene (PTFE) plaque. A preliminary evaluation indicates that the expanded uncertainty ( $k=2$ ) of the reference spectroradiometer in measuring  $x$ ,  $y$  values of these CRT colors was 0.001 and that in  $Y$  values 0.4 %, with respect to the standard source. The details of this uncertainty budget are beyond the scope of this paper and will be a subject of a future publication. The uncertainties of the target instrument in measuring  $x$ ,  $y$  and  $Y$  are unknown but are not relevant to this experiment. The repeatability of the target instrument in measuring the CRT was 0.001 in  $x$ ,  $y$  and less than 0.5 % in  $Y$ .

From the data shown in Table 2, the corrected ( $x$ ,  $y$ ) values for the target instrument for all colors were computed by using four methods: (1) RGB method, (2) ASTM-92, (3) ASTM-96, and (4) Four-Color method. The data for the first 8 colors, which included primary colors and white, were used to compute the correction matrices for the ASTM methods. The other 6 colors were measured as test colors. The differences ( $x$ ,  $y$ ) between the corrected chromaticity values and those measured with the reference instrument shown in Table 2 were calculated. Also, the same computation was conducted for the set of data published in reference [2] which consist of 18 CRT colors measured by a target instrument and a reference instrument. Figures 2 and 3

show the rms values and the maximum values, respectively, of the differences ( $x$ ,  $y$ ) of all 14 or 18 colors after corrections using the four methods. The figures also show the original errors (the differences between the  $x$ ,  $y$  values of the target instrument and those of the reference instrument shown in Table 2). These data demonstrate a considerable improvement of chromaticity accuracy with the Four-Color Method over other methods.

## 5. CONCLUSION

A new method (Four-Color Method) has been developed to improve the accuracy of chromaticity measurements of displays using tristimulus colorimeters. This method is independent of  $Y$  values, thus eliminating errors due to luminance measurement errors. The analysis using two sets of experimental data demonstrated a considerable improvement with this method over conventional methods. In addition, this new method has an advantage of simplicity when compared with the ASTM-96 method.

Each set of experimental data was taken with one particular CRT. Further study is required to show how sensitive this method might be to displays employing different phosphor sets having different spectral power distributions.

Luminance values ( $Y$ ) can also be corrected by utilizing the  $R$  matrix obtained with this method. The matrix should be multiplied by a scaling factor that allows the computed  $Y$  values to match the luminance values of the four colors measured by a reference instrument. The accuracy of this luminance correction compared with other conventional methods are being studied.

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## References

1. ASTM E 1455-92, Standard Practice for Obtaining Colorimetric Data from a Visual Display Unit Using Tristimulus Colorimeters (1992).
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3. C. Leone, S. Sojourner, E. Vargas, C. Cromer, Y. Ohno, and J. Hardis, Improved Chromaticity and Luminance Measurements Using a Tristimulus Colorimeter, Proc. SID 96, 433-436 (1996).

Table 1. The calculated chromaticity coordinates and luminance values of the 16 colors

CRT Color	True values			Model colorimeter			Random noise(%)	Y (cd/m <sup>2</sup> ) with noise
	x	y	Y (cd/m <sup>2</sup> )	x	y	Y (cd/m <sup>2</sup> )		
Color 1 (White)	0.2866	0.2954	197.44	0.2872	0.3002	198.10	-1.9	194.34
Color 2 (Red)	0.5953	0.3449	48.04	0.5957	0.3434	46.22	0.6	46.48
Color 3 (Green)	0.2733	0.5853	125.99	0.2672	0.5907	129.05	-2.1	126.29
Color 4 (Blue)	0.1587	0.0746	23.41	0.1661	0.0744	22.83	-1.9	22.38
Color 5 (Yellow)	0.3998	0.4908	174.03	0.3924	0.4964	175.27	1.5	177.95
Color 6 (Cyan)	0.2053	0.2823	149.40	0.2081	0.2891	151.88	-2.7	147.82
Color 7 (Magenta)	0.2929	0.1576	71.45	0.2971	0.1564	69.05	2.2	70.59
Color 8	0.2095	0.1932	82.23	0.2138	0.1969	82.50	-1.4	81.33
COLOR 9	0.3743	0.2689	81.92	0.3748	0.2703	80.48	0.6	80.95
COLOR 10	0.3051	0.4508	149.42	0.3012	0.4576	151.65	-0.3	151.15
COLOR 11	0.2286	0.1630	66.95	0.2332	0.1646	66.26	-0.1	66.19
COLOR 12	0.4058	0.3577	102.04	0.4032	0.3612	101.31	-1.3	100.02
COLOR 13	0.2588	0.3954	144.59	0.2576	0.4027	147.07	-2.3	143.74
COLOR 14	0.2892	0.2397	52.72	0.2911	0.2426	52.38	2.6	53.72
COLOR 15	0.3258	0.3631	72.84	0.3239	0.3686	73.21	-0.5	72.87
COLOR 16	0.2507	0.2896	68.01	0.2521	0.2953	68.62	-1.9	67.34

Table 2. The chromaticity coordinates and luminance values measured by the reference instrument and the target instrument.

CRT Color	Reference instrument			Target instrument		
	x	y	Y (cd/m <sup>2</sup> )	x	y	Y (cd/m <sup>2</sup> )
Color 1 (White)	0.3148	0.3173	129.2	0.316	0.328	116
Color 2 (Red)	0.6322	0.3362	65.25	0.620	0.348	61.1
Color 3 (Green)	0.3078	0.5930	183.5	0.314	0.586	169.
Color 4 (Blue)	0.1500	0.0623	22.46	0.143	0.062	19.4
Color 5 (Yellow)	0.4315	0.4965	149.9	0.428	0.499	140
Color 6 (Cyan)	0.2244	0.3107	142.3	0.228	0.320	134
Color 7 (Magenta)	0.3200	0.1592	58.70	0.318	0.166	56.2
Color 8	0.1741	0.1238	50.87	0.171	0.128	46.5
Color 9	0.5319	0.2936	67.53	0.525	0.305	64.2
Color 10	0.3251	0.5599	175.0	0.329	0.557	167
Color 11	0.3146	0.3170	128.6	0.315	0.327	124
Color 12	0.2490	0.3155	32.51	0.251	0.326	31.1
Color 13	0.3101	0.2014	17.11	0.309	0.211	16.0
Color 14	0.3911	0.4436	36.90	0.391	0.449	34.7